RADIAL CORRELATIONS BETWEEN TWO QUARKS

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In nuclear *many*-body problems the short-range correlation between two nucleons is well described by the corresponding correlation in the *two*-body problem. Therefore, as a first step in any attempt at an analogous description of *many-quark* systems, it is necessary to know the *two-quark* correlation. With this in mind, we study the light quark distribution in a heavy-light meson with a static heavy quark. The charge and matter radial distributions of these heavy-light mesons are measured on a lattice with a light quark mass about that of the strange quark. Both distributions can be well fitted upto $r \approx 0.7$ fm with the exponential form $w_i^2(r)$, where $w_i(r) = A\exp(-r/r_i)$. For the charge(c) and matter(m) distributions $r_c \approx 0.32(2)$fm and $r_m \approx 0.24(2)$fm. We also discuss the normalisation of the total charge (defined to be unity in the continuum limit) and matter integrated over all space, finding 1.30(5) and 0.4(1) respectively for a lattice spacing ≈ 0.17 fm.

1 Correlations between two nucleons

In nuclear many-body problems it is well known that the short-range correlations between pairs of nucleons are often well described by the corresponding correlations of the two-body problem. For example:

- **Few nucleon systems.** The NN correlations for the deuteron, ^3He and ^4He, when normalised to each other at their maxima, are essentially indistinguishable upto about 2 fm. Only for larger internucleon distances do the binding energy differences play a role [1].

- **Many-nucleon systems.** Often variational methods are used and a typical trial wavefunction is taken to be of the form

$$|\Psi_T> = [\Pi_{i<j} F_{ij}]\,|\Psi(\text{Shell Model})>, \qquad (1)$$

where Ψ(Shell Model) is simply a product(antisymmetrised) of single nucleon wavefunctions – encoding the long range correlations. However,

the short range physics between the NN pairs is in the

$$F_{ij} = \left[1 + \sum_m u_m(r_{ij}) O_{ij}^m\right], \tag{2}$$

where $O_{ij}^m = [1, \sigma_i.\sigma_j, ...] \bigotimes [1, \tau_i.\tau_j]$ are the various spin and isospin operators for two nucleons. The variation then yields radial correlations $u_m(r_{ij})$ that are $\approx$ 2-body problem correlations – see for example Ref. [2].

- **Nuclear Matter** An extreme use in many-body systems of the two-body correlation is the Moszkowski-Scott separation method[3] in nuclear matter. There the many-body wavefunction is taken to be of the form

$$\begin{aligned} \Psi(\text{Many} - \text{Body}) &= \Psi(\text{Two} - \text{Body}) \quad r < r_c \\ &= \sin(kr) \qquad\qquad\quad r \geq r_c, \end{aligned} \tag{3}$$

where r_c is the value of r at which $\Psi(\text{Two} - \text{Body})$ and $\sin(kr)$ and their derivatives equal each other.

The conclusion is that, in many-nucleon systems, the two-nucleon correlation from the two-body problem plays an important role.

The question can then be asked whether similar effects arise in multi-*quark* systems. This may help in the understanding of such systems. At present and in the forseeable future, since lattice QCD calculations are restricted to very few quarks (upto four), some model *that encodes the results of these few-quark calculations* is needed in order to proceed to systems containing more quarks.

Earlier attempts by the present authors have concentrated on understanding the *energies* of four-quark systems in terms of interquark potentials. These involved four heavy quarks $(Q^2\bar{Q}^2)$[4] and the interaction of two heavy-light mesons $(Q^2\bar{q}^2)$ [5]. In neither case was it possible to understand the four-quark energies in terms of simply two-quark potentials – with the result indicating that a four-quark form-factor was necessary. This conclusion has also been supported by colour field measurements [6]. Here, as a first step, the emphasis is on extracting the *radial correlation* between the two quarks of a heavy-light meson. Hopefully, this additional information on the $Q\bar{q}$ system will enable a less ambiguous model of this system to be made. At present, if through an operator $O(r)$, a transition between two two-quark states is calculated as $\langle\psi_1(r)|O(r)|\psi_2(r)\rangle$, then the ψ_i are first calculated as a byproduct of some differential equation containing some interquark potential - say $V(r) = a/r + br$. However, both the form of $V(r)$ and the equation into which it is inserted are not unique. The latter could range from a non-relativistic Schroedinger equation, if the quarks are heavy enough, through a series of equations that incorporate relativistic effects to varying degrees.

2 Heavy-light mesons ($Q\bar{q}$)

A study of heavy-light mesons is not just for the academic interest discussed above, since they are realised in nature. The best examples are the $B(5.28$ GeV) and $B_s(5.37$ GeV) mesons, which have quark structures $(\bar{b}u)$ and$(\bar{b}s)$. Since the b, s and u-quarks have masses of about 4.2, 0.1 and 0.001 GeV, we see that the B and B_s are indeed heavy-light mesons and can be thought of as the "Hydrogen atom" for quark systems. Currently the B-mesons are of particular interest, since they are expected to lead to a better understanding of CP violation and for this reason are being generated at so-called B-factories[7].

More details of the following sections can be found in Ref.[8].

2.1 Energies of Heavy-light mesons

The basic quantity for evaluating the energies of heavy-light mesons is the 2-point correlation function – see Fig. 1 a).

$$C(2,T) = \langle U^Q(\mathbf{x},t,T)P(\mathbf{x},t+T,t)\rangle, \tag{4}$$

where $U^Q(\mathbf{x},t,T)$ is the heavy(infinite mass)-quark propagator and $P(\mathbf{x},t+T,t)$ the light-quark propagator. The $\langle ...\rangle$ means that $C(2)$ has been averaged over the whole lattice. Since the $C(2)$ decay as $\exp(-E_0T)$, where E_0 is the energy of the ground state, we get

$$E_0 = -\ln\,[\frac{\langle C(2,T)\rangle}{\langle C(2,T-1)\rangle}] \quad \text{as } T\to\infty. \tag{5}$$

These energies were calculated in Ref.[9] for the states

$L_J = S_{1/2},\ P_{1/2},\ P_{3/2},\ D_{3/2},\ D_{5/2},\ F$

2.2 Heavy-Light radial correlations

Here the basic quantity is 3-point correlation function – see Fig. 1 b).

$$C(3,-t_2,\ t_1,\ \mathbf{r}) = \langle U^Q(\mathbf{x},-t_2,\ t_1)P_1(\mathbf{x},t_1;\mathbf{r},0)\Theta(\mathbf{r})P_2(\mathbf{r},0;\mathbf{x},-t_2)\rangle, \tag{6}$$

where the $P_{1,2}$ are the light anti-quark propagators that go from the Q at time t_1 to the point $\mathbf{r}$ at $t=0$ and then return to Q at time $-t_2$. The probe $\Theta(\mathbf{r})$ at $\mathbf{r}$ is here considered to have two forms i) $\Theta=\gamma_4$ for measuring the charge distribution of the $\bar{q}$ and ii) $\Theta=1$ for measuring the matter distribution.

Knowing the $C(3)$ then the radial distributions are given by

$$F[C(\Theta,T,R)] = \frac{\langle C(3,\Theta,T,R)\rangle}{\langle C(2,T)\rangle}. \tag{7}$$

2.3 Lattice Parameters and Refinements

The essential lattice parameters are:

1) Lattice size $16^3 \times 24$

2) Quark-gluon coupling $\beta = 5.7$ giving a lattice spacing ≈ 0.17fm

3) Light quark mass $\approx m_s$ – see Ref. [11] as evidence for this.

4) Quenched Approximation i.e. quark-antiquark pairs are not included.

Eqs. 5 and 7 for the ground state energy(E_0) and correlations $F[C(\Theta, T, R)]$ are based on a single state constructed from the basic lattice. However, this can be extended by modifying this original lattice to give other states $(i, j, k...)$. In this way the 2- and 3-point correlation functions can become matrices $C_{i,j}(2)$, $C_{i,j}(3, \mathbf{r})$. Their diagonalisation – by reducing the contamination of higher states on the ground state – then gives improved estimates of E_0 and $F[C(\Theta, T, R)]$. The mechanism for generating these addition states is fuzzing. This replaces a link on the lattice by a combination of neighbouring links to give a fuzzed link – with projection to SU3 implied – as

[A fuzzed link] = f_p·[Straight link]+ [Sum of 4 spatial U-bends].

This cycle can be repeat many times. Here, in addition to the basic state(L), two new states F1 and F2 are constructed using 2 and 6 cycles with $f_p = 2.5$. respectively.

3 Results

The lattice data $\langle C_{i,j}(2, T)\rangle$ and $\langle C_{i,j}(3, T, \mathbf{r})\rangle$ can be analysed in several ways. Here two are presented: i) Visual and ii) Fitting with exponentials.

3.1 Visual

Fig. 2 shows from Eq. 7 the ratio $\langle C(3, R)\rangle / \langle C(2)\rangle$ for values of r upto 5 lattice spacings i.e. about 0.8fm. As $T \to \infty$, these clearly show plateaux, which give directly the desired charge or matter density.

3.2 Fitting with exponentials

A more precise method is to first fit the $\langle C_{i,j}(2, T)\rangle$ by the approximate expression

$$\tilde{C}_{ij}(2, T) = \sum_{\alpha=1}^{3} v_i^\alpha \exp(-E_\alpha T) v_j^\alpha, \tag{8}$$

where the i, j refer to the states L,F1,F2. This results in the energy eigenvalues E_α and their eigenvectors v_i^α. These E_α are the energies quoted in Ref. [9]. It

Table 1. The parametrization of the charge and matter densities as $w_m^2(r)$, where $w_m(r) = A_m \exp[-r/r_m]$.

	A_i	r_i (fm)	RMS Radius (fm)
Charge Density	0.26(1)	0.32(2)	0.55(3)
Matter Density	0.29(1)	0.24(2)	0.42(3)

is found that only data with $T \geq 4$ gives $\chi^2/dof \approx 1$. This means that 54 data points are fitted with 12 parameters.

Given the E_α and v_i^α the $C_{ij}(3,T,R)$ are then fitted by the approximate expression

$$\tilde{C}_{ij}(3,T,r) = \sum_{\alpha=1}^{3}\sum_{\beta=1}^{3} v_i^\alpha \exp[-E_\alpha t_1] x^{\alpha\beta}(r) \exp[-E_\beta t_2] v_j^\beta, \qquad (9)$$

where the $x^{\alpha\beta}(r)$ are varied giving directly the desired charge or matter density. This needs $t_1 + t_2 \geq 8$ to get $\chi^2/dof \approx 1$ when fitting 18 data points with 3 parameters.

3.3 Analytic forms for the charge and matter densities

For the above charge (i=c) and matter ($i = m$) densities, it is of interest to express $x_i(r) = w_i^2(r)$ in the form $w_i(r) = A_i \exp(-r/r_i)$, where the parameters r_i acd A_i are given for a global fit in Table 3.3. There, as in Fig. 3, it is clearly seen that the charge density has a significantly longer range than that of the matter density.

The fourier transforms of these densities lead to vector(i=$v = c$) and scalar($i = s = m$) form factors that are usually expressed as $[F_{v,s}(q^2)] \propto (q^2 + M_{v,s}^2)^{-1}$. These are appropriate for the long-range part of $w_i^2(r)$ and lead to M_v=0.9(1) and $M_s = 1.3(1)$GeV and should be compared with the the direct calculation of the $s\bar{s}$ vector and scalar mesons in Refs. [10] and [11]. The latter only involve 2-point correlation functions and so are more precise leading to M_v=0.944(2) and $M_s = 1.61(6)$GeV.

3.4 Sum-rules

The above measures the densities at a few definite values of r. However, it is of interest to consider the sum-rules that sum over all values of r i.e.

$$F^{\rm sum}[C(3)] = \frac{\langle \sum_{\mathbf{r}} C(3,\mathbf{r}) \rangle}{\langle C(2) \rangle}. \qquad (10)$$

For the charge and matter densities this yields I_c=1.30(5) and I_m=0.4(1) respectively. On the other hand, integrating the above analytic expressions for $w_i(r)$ gives 1.5(1) and 0.7(1). These estimates are less reliable than the direct lattice sum, since much of the contributions are from small values of r where lattice artifacts enter.

By charge conservation we should get I_c=1 in the continuum limit – the quark charge having been defined as unity. However, it is known that renormalisation effects enter due to the finite lattice spacing. In Ref. [12], for lattice parameters different to those used here, an overall renormalisation factor (F^V) of ≈ 0.8 is found for the vector(charge) vertex. Similar reductions are found in Ref. [13] for the axial vector operator. The inverse of our value of I_c=1.30(5) could then be interpreted as a non-perturbative estimate of F^V as 0.77(3). In any case the conclusion is that our I_c=1.30(5) is consistent with charge conservation for the continuum.

3.5 Dirac amplitude interpretation

In a relativistic quantum mechanical picture the above charge and matter distributions can be interpreted as $|g|^2+|f|^2$ and $|g|^2-|f|^2$, where g, f are the upper and lower components in the solution of the Dirac equation. Possibly this will now serve as a way to remove some of the ambiguities refered to at the end of Section 1 by ensuring that the solutions of the Dirac equation agree not only with the lattice or empirical energies but also the forms of $|g(r)|^2$ and $|f(r)|^2$.

Conclusions and Future

- The $S_{1/2}$-wave charge and matter densities can be measured quite reliably out to $\approx$ 0.7fm.
- Off-axis measurements can improve these results. In particular, it is interesting to see whether or not the charge density for $r = 5$ is indeed lower than the simple exponential drop off in Fig. 3. This would indicate that the confining potential is playing a role. Hopefully, the better statistics at the off-axis point $(3, 4)$ would clarify this point.
- The $P_{1/2},\ P_{3/2},\ D_{3/2},\ D_{5/2}, ...$ densities are now being measured.
- Understand the densities phenomenologically using the Dirac equation.
- Measure correlations in the $(Q^2\bar{q}^2)$ system and check their form against the above $Q\bar{q}$-correlations.

- Use larger β and lattices to get nearer the continuum limit.
- Replace the present quenched lattices by unquenched ones.
- Consider other operators, e.g. Pseudo-vector ($\gamma_\mu\gamma_5$) for $B^*B\pi$ coupling.

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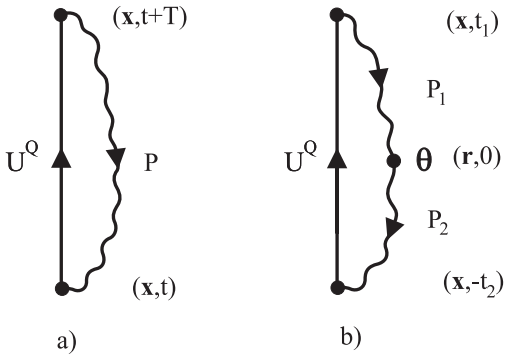


Figure 1. a) A two-point correlation function, b) A three-point correlation function.

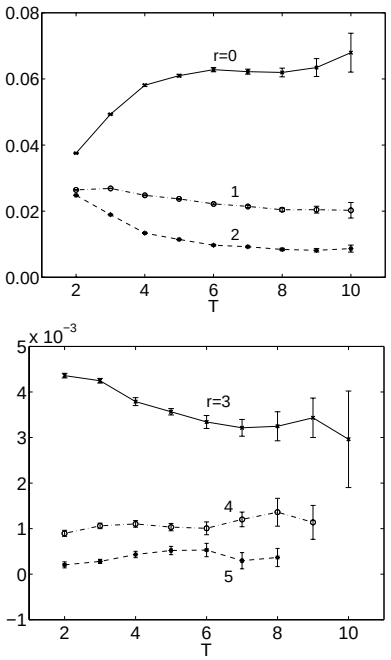


Figure 2. The ratio $\langle C(3,r)\rangle/\langle C(2)\rangle$ for $r = 0, \ldots, 5$.

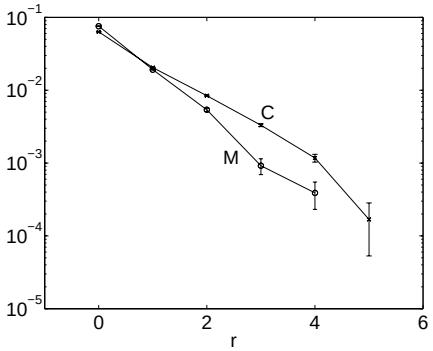
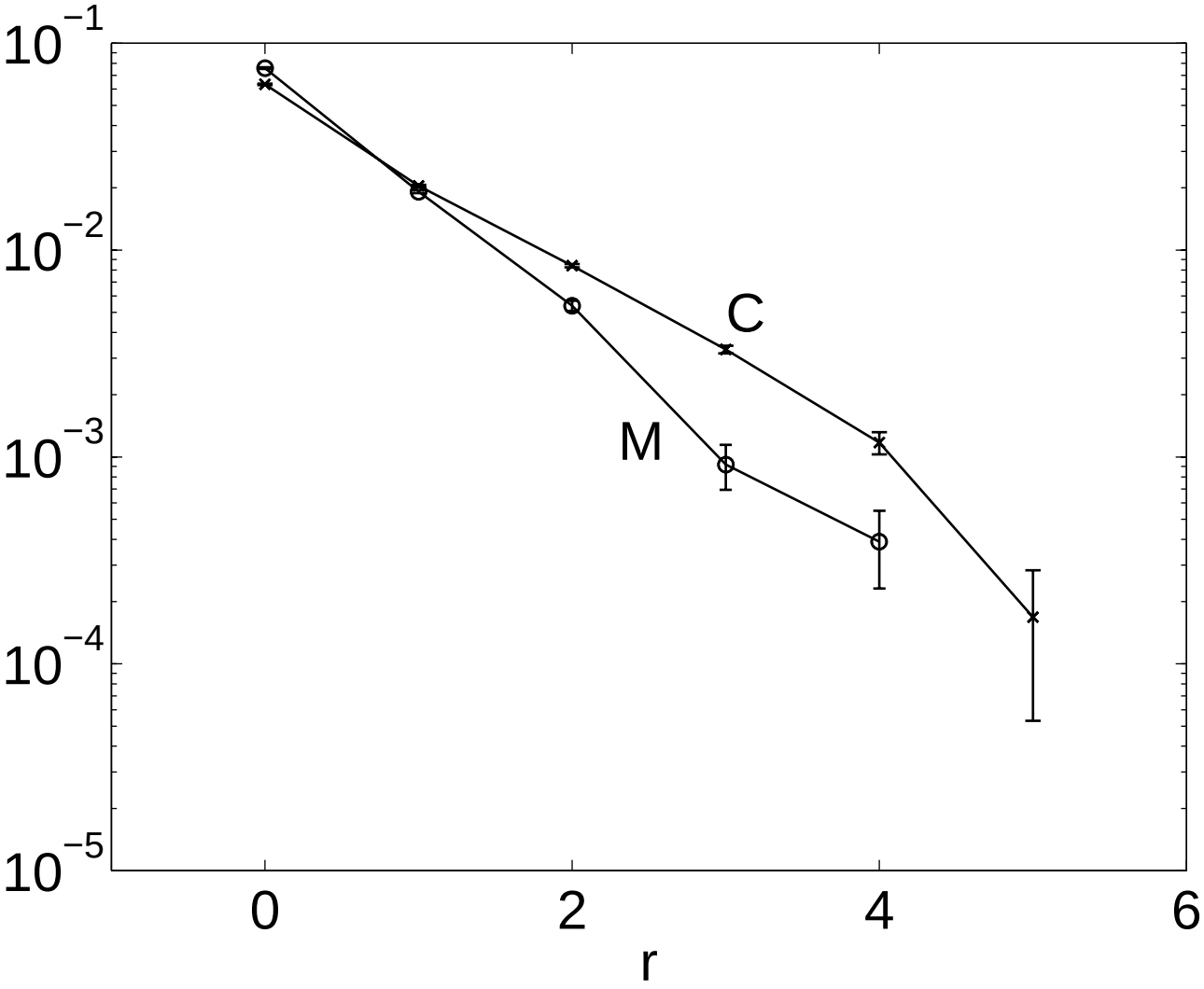


Figure 3. The radial distribution of the ground state charge(C) and matter(M) densities.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